**Proposed Method as an Average of Grouped Base Conditions**

Development of the proposed method starts from the presumption that only those runs in which an activity is either fully on or fully off can be used. To avoid bias, all possible such runs should be considered. Given four target stresses, notated *C, K,* and *N* for Colorado, Kansas, and Nebraska pumping and *M* for mound recharge, there are 16 possible model runs with each stress either on or off. Using the presence of the letter in the run name to indicate that the corresponding stress in on, these are:

*θ, C, K, M, N, CK, CM, CN, KM, KN, MN, CKM, CKN, CMN, KMN, CKMN*

with *θ* representing the run with all stresses off. Each run will produce computed baseflow at a given accounting point in a given year. With these 16 runs and their associated computed baseflows, there are eight ways to evaluate the change in baseflow with the addition of a target activity. For example, the impact of Nebraska pumping can be evaluated by any one of these eight differences:

*θ-N, C-CN, K-KN, M-MN, CK-CKN, CM-CMN, KM-KMN, CKM-CKMN*

where the symbols now indicate the value of baseflow that results from the indicated run. Each of these eight differences gives information, from the model, on how baseflow responds to Nebraska pumping. For this discussion, the first value in each difference will be considered the base condition. These eight differences can be viewed as shown in table 1 where the differences are grouped by the number of human activities in the on condition for a base run.

**Table 1: Grouping of eight differences based on the number of activities in the base condition.**

| **Base Condition** | **Resulting Run Differences** |
|---|---|
| no human activity | *θ-N* |
| one activity on (*C, K* or *M*) | *C-CN, K-KN, M-MN* |
| two activities on (*CK, CM* or *KM*) | *CK-CKN, CM-CMN, KM-KMN* |
| three activities on (*CKM*) | *CKM-CKMN* |

Given the four groups of base types in table 1, the proposed method takes the average difference within each group and then the average over all groups. That is, the average for the differences with one activity on is

$(C\text{-}CN + K\text{-}KN + M\text{-}MN)/3$

and the average of those differences that have two activities on is

$(CK\text{-}CKN + CM\text{-}CMN + KM\text{-}KMN)/3$

So that the average over all differences is ¼ of the differences in each group as follows:

$$
\begin{aligned}
CBCU_N = [&\theta\text{-}N \\
&+(C\text{-}CN + K\text{-}KN + M\text{-}MN)/3 \\
&+ (CK\text{-}CKN + CM\text{-}CMN + KM\text{-}KMN)/3 \\
&+ CKM\text{-}CKMN]/4
\end{aligned}
$$

The proposed method can be viewed as given the same weight to each level of difference. Evaluating Nebraska pumping with no other stresses on (i.e., $\theta\text{-}N$), is the first level, evaluating Nebraska pumping with one other stress on ($C\text{-}CN$, $K\text{-}KN$, $M\text{-}MN$) is the next level, and so on.

**Proposed Method Will Always Have Zero Residual**

The residual has been defined as the difference between the independently-calculated *VWS* associated with groundwater, $VWS_G$, and the sum of computed impacts as follows

$$R = VWS_G - (CBCU_C + CBCU_K + CBCU_N - IWS)$$

Using the definition of $VWS_G$ as the difference in baseflows with all activities on and all activities off, it can be defined as

$$VWS_G = \theta\text{-}CKMN$$

Substituting this definition and the equations for the proposed method in the equation for the residual results in:

$$
\begin{aligned}
R = {} & \theta\text{-}CKMN \\
& - [(\theta\text{-}C) + ((K\text{-}CK) + (M\text{-}CM) + (N\text{-}CN))/3 + \\
& \qquad ((KM\text{-}CKM) + (KN\text{-}CKN) + (MN\text{-}CMN))/3 + (KMN\text{-}CKMN)]/4 \\
& + [(\theta\text{-}K) + ((C\text{-}CK) + (M\text{-}KM) + (N\text{-}KN))/3 + \\
& \qquad ((CM\text{-}CKM) + (CN\text{-}CKN) + (MN\text{-}KMN))/3 + (CMN\text{-}CKMN)]/4 \\
& + [(\theta\text{-}N) + ((C\text{-}CN) + (M\text{-}MN) + (K\text{-}KN))/3 + \\
& \qquad ((CM\text{-}CMN) + (CK\text{-}CKN) + (KM\text{-}KMN))/3 + (CKM\text{-}CKMN)]/4 \\
& - [(M\text{-}\theta) + ((CM\text{-}C) + (KM\text{-}K) + (MN\text{-}N))/3 + \\
& \qquad ((CKM\text{-}CK) + (CMN\text{-}CN) + (KMN\text{-}KN))/3 + (CKMN\text{-}CKN)]/4
\end{aligned}
$$

Examination of this equation shows that the right side reduces to zero, indicating that the values of $CBCU_C$, $CBCU_K$, $CBCU_N$, and $IWS$ produced by the proposed method will always give zero residual.

**Weights On Proposed Method Insure Zero Residual**

The fact that the proposed method produces values of impacts that always sum to zero is a result of the weights selected for the eight differences. Considering again the case of Nebraska pumping, the equation for $CBCU_N$ can be written as a general weighted combination of individual differences as

$$CBCU_N = x_1(\theta\text{-}N) + x_2(C\text{-}CN) + x_3(M\text{-}MN) + x_4(K\text{-}KN) + x_5(CM\text{-}CMN) + x_6(CK\text{-}CKN) + x_7(KM\text{-}KMN) + x_8(CKM\text{-}CKMN)$$

where $x_n$ represents the weights and the summation of the weights must equal one. If the baseflow response is linear then all of the eight differences will be identical and the weights are unimportant. They are critical, however, when the baseflow response is nonlinear. Note that the current method assigns a weight of one to $x_8$ and zero to all other weights. A similar equation can be written for the other three target stresses using the same eight weights.

In the proposed method, the weights take values that insure that the residual will always be zero. This can be seen by setting the sum of the impacts of the four target stresses equal to the total impact as follows

$$\begin{aligned}
&[x_1(\theta\text{-}C) + x_2(K\text{-}CK) + x_3(M\text{-}CM) + x_4(N\text{-}CN) + \\
&\qquad x_5(KM\text{-}CKM) + x_6(KN\text{-}CKN) + x_7(MN\text{-}CMN) + x_8(KMN\text{-}CKMN)] + \\
&[x_1(\theta\text{-}K) + x_2(C\text{-}CK) + x_3(M\text{-}KM) + x_4(N\text{-}KN) + \\
&\qquad x_5(CM\text{-}CKM) + x_6(CN\text{-}CKN) + x_7(MN\text{-}KMN) + x_8(CMN\text{-}CKMN)] + \\
&[x_1(\theta\text{-}N) + x_2(C\text{-}CN) + x_3(M\text{-}MN) + x_4(K\text{-}KN) + \\
&\qquad x_5(CM\text{-}CMN) + x_6(CK\text{-}CKN) + x_7(KM\text{-}KMN) + x_8(CKM\text{-}CKMN)] + \\
&[x_1(\theta\text{-}M) + x_2(C\text{-}CM) + x_3(K\text{-}KM) + x_4(N\text{-}MN) + \\
&\qquad x_5(CK\text{-}CKM) + x_6(CN\text{-}CMN) + x_7(KN\text{-}KMN) + x_8(CKN\text{-}CKMN)] \\
&\qquad = (\theta\text{-}CKMN)
\end{aligned}$$

Where the same weights have been applied to each impact equation and the final group, representing $IWS$, has been multiplied through by minus one to simplify the analysis. The best values for each weight ($x_n$) can be determined by examination of the equation. The runs $\theta$ and *CKMN* (with a positive sign and negative sign, respectively) each occur four times on the left side of the equation, and only once on the right side of the equation. So $x_1$ and $x_8$ must equal ¼. The runs *C, K, M,* and *N* occur once with a negative sign and three times with a positive sign on the left side, and they do not occur on the right side. The negative terms already have a weight of ¼, so each positive term must have a weight of $^1/_{12}$ so that they cancel. So $x_2$, $x_3$, and $x_4$ must equal $^1/_{12}$. Each run with two stresses on and two stresses off occur twice as a positive (after $x_5$, $x_6$, or $x_7$) and twice as a negative (after $x_2$, $x_3$, or $x_4$), meaning these two sets of weights must be equal. So:

$$x_1 = x_2 = 1/4 \quad \text{and} \quad x_2 = x_3 = x_4 = x_5 = x_6 = x_7 = {}^1/_{12}$$

These weights also ensure that the three negative occurrences of *CKM, CKN, KMN*, and *CMN* (with a weight of $^1/_{12}$) cancel the one positive occurrence of each (with a weight of ¼). Assigning these weights produces the proposed method.

**Proposed Method Developed From Sequential Subtraction of Impacts**

The proposed method can be developed from sequential subtraction of impacts. Begin by expanding the equation ($\theta$-*CKMN*) to include an evaluation of each target stress. For example:

$$\theta\text{-}CKMN = \theta\text{-}C + C\text{-}CK + CK\text{-}CKM + CKM\text{-}CKMN$$

The right side of this equation includes four differences. The first gives the response of baseflow with change in Colorado pumping when the all-off base is used. The second gives the response of baseflow when Kansas pumping is activated from a Colorado-on base condition. The third and fourth differences provide estimates of mound and Nebraska impacts from different base conditions. The four terms on the right side of this equation could be used to define $CBCU_C$, $CBCU_K$, *IWS,* and $CBCU_N$, respectively, and these impact values would be guaranteed to have zero residual. However, there is obvious bias in selecting the order in which the activities are subtracted. Other orders of subtraction are possible. Indeed, there are a total of 24 such combinations[1] and they are listed here:

$$\theta\text{-}CKMN = \theta\text{-}C + C\text{-}CK + CK\text{-}CKM + CKM\text{-}CKMN$$

$$\theta\text{-}CKMN = \theta\text{-}C + C\text{-}CK + CK\text{-}CKN + CKN\text{-}CKMN$$

[1] These are all possible permutations of this sequence. The maximum number of permutations for a given sequence is equal to the factorial of the length of the sequence (in this case 4! = 24).

$\theta\text{-}CKMN = \theta\text{-}C + C\text{-}CN + CN\text{-}CKN + CKN\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}C + C\text{-}CN + CN\text{-}CMN + CMN\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}C + C\text{-}CM + CM\text{-}CKM + CKM\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}C + C\text{-}CM + CM\text{-}CMN + CMN\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}K + K\text{-}CK + CK\text{-}CKM + CKM\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}K + K\text{-}CK + CK\text{-}CKN + CKN\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}K + K\text{-}KM + KM\text{-}CKM + CKM\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}K + K\text{-}KM + KM\text{-}KMN + KMN\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}K + K\text{-}KN + KN\text{-}CKN + CKN\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}K + K\text{-}KN + KN\text{-}KMN + KMN\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}M + M\text{-}CM + CM\text{-}CKM + CKM\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}M + M\text{-}CM + CM\text{-}CKM + CKM\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}M + M\text{-}KM + KM\text{-}CKM + CKM\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}M + M\text{-}KM + KM\text{-}KMN + KMN\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}M + M\text{-}MN + MN\text{-}CMN + CMN\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}M + M\text{-}MN + MN\text{-}KMN + KMN\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}N + N\text{-}CN + CN\text{-}CKN + CKN\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}N + N\text{-}CN + CN\text{-}CMN + CMN\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}N + N\text{-}KN + KN\text{-}CKN + CKN\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}N + N\text{-}KN + KN\text{-}KMN + KMN\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}N + N\text{-}MN + MN\text{-}CMN + CMN\text{-}CKMN$

$\theta\text{-}CKMN = \theta\text{-}N + N\text{-}MN + MN\text{-}KMN + KMN\text{-}CKMN$

The proposed method can be viewed as averaging over all 24 of these possible sequences.

Summing these 24 equations and rearranging yields:

$$[(\theta\text{-}C) + ((K\text{-}CK) + (M\text{-}CM) + (N\text{-}CN))/3 +$$
$$((KM\text{-}CKM) + (KN\text{-}CKN) + (MN\text{-}CMN))/3 + (KMN\text{-}CKMN)]/4 +$$
$$[(\theta\text{-}K) + ((C\text{-}CK) + (M\text{-}KM) + (N\text{-}KN))/3 +$$
$$((CM\text{-}CKM) + (CN\text{-}CKN) + (MN\text{-}KMN))/3 + (CMN\text{-}CKMN)]/4 +$$
$$[(\theta\text{-}M) + ((C\text{-}CM) + (K\text{-}KM) + (N\text{-}MN))/3 +$$
$$((CK\text{-}CKM) + (CN\text{-}CMN) + (KN\text{-}KMN))/3 + (CKN\text{-}CKMN)]/4 +$$
$$[(\theta\text{-}N) + ((C\text{-}CN) + (M\text{-}MN) + (K\text{-}KN))/3 +$$

$$((CM\text{-}CMN) + (CK\text{-}CKN) + (KM\text{-}KMN))/3 + (CKM\text{-}CKMN)]/4 = (\theta\text{-}CKMN)$$

The four groupings on the left side of the equation can be recognized as the equations for $CBCU_C$, $CBCU_K$, $IWS$ and $CBCU_N$, respectively, in the proposed method (again with $IWS$ multiplied by minus one to simplify the analysis). The four computed impacts are guaranteed to have zero residual and remove bias from the selection of base condition.